MODFLOW: A Finite-Difference Groundwater Flow Model or an Integrated Finite-Difference Groundwater Flow Model?

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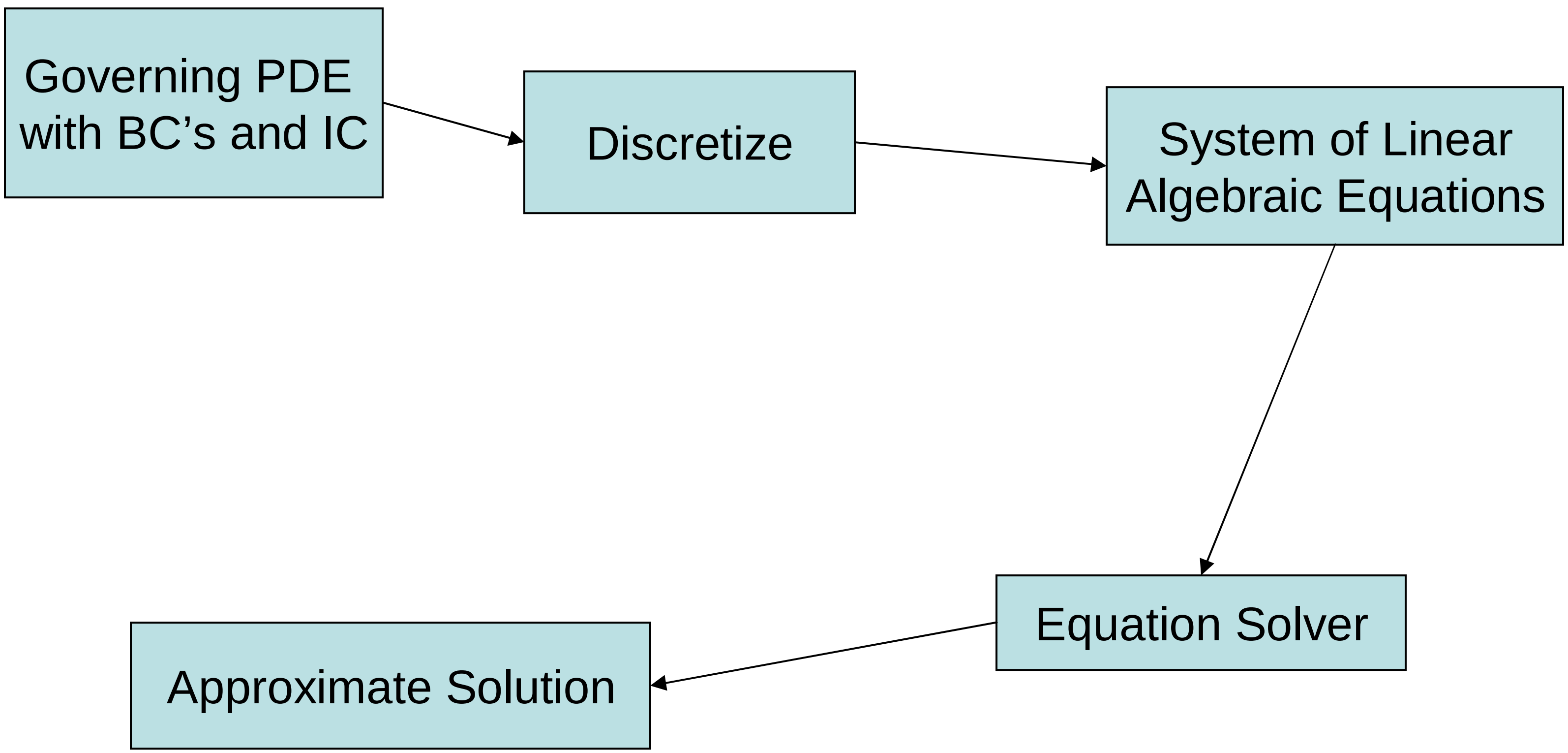
DISCUSSION TOPICS

- Discretization Processes
  - Finite Difference
  - Integrated Finite Difference
  - MODFLOW
- MODFLOW Modifications
- Test Problem

FINITE-DIFFERENCE DISCRETIZATION

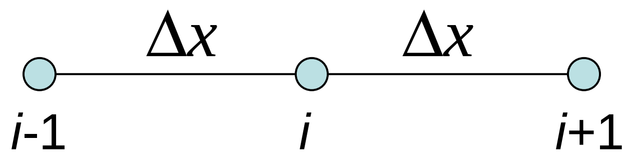


Governing Flow Equation

$$\frac{\partial}{\partial x}\left(K_x \frac{\partial h}{\partial x}\right)+\frac{\partial}{\partial y}\left(K_y \frac{\partial h}{\partial y}\right)+\frac{\partial}{\partial z}\left(K_z \frac{\partial h}{\partial z}\right)+q=S_s \frac{dh}{dt} \qquad \left[\frac{1}{T}\right]$$

Homogenous Form

$$K_x \frac{\partial^2 h}{\partial x^2}+K_y \frac{\partial^2 h}{\partial y^2}+K_z \frac{\partial^2 h}{\partial z^2}+q=S_s \frac{dh}{dt} \qquad \left[\frac{1}{T}\right]$$



First Derivative

$$\frac{\partial h}{\partial x} \cong \frac{h_{i-1} - h_i}{\Delta x}$$

Second Derivative

$$\frac{\partial^2 h}{\partial x^2} = \frac{\partial}{\partial x}\left(\frac{\partial h}{\partial x}\right) \cong \frac{\frac{h_{i-1} - h_i}{\Delta x} - \frac{h_i - h_{i+1}}{\Delta x}}{\Delta x} = \frac{h_{i-1} - 2h_i + h_{i+1}}{(\Delta x)^2}$$

Continuous Governing Equation

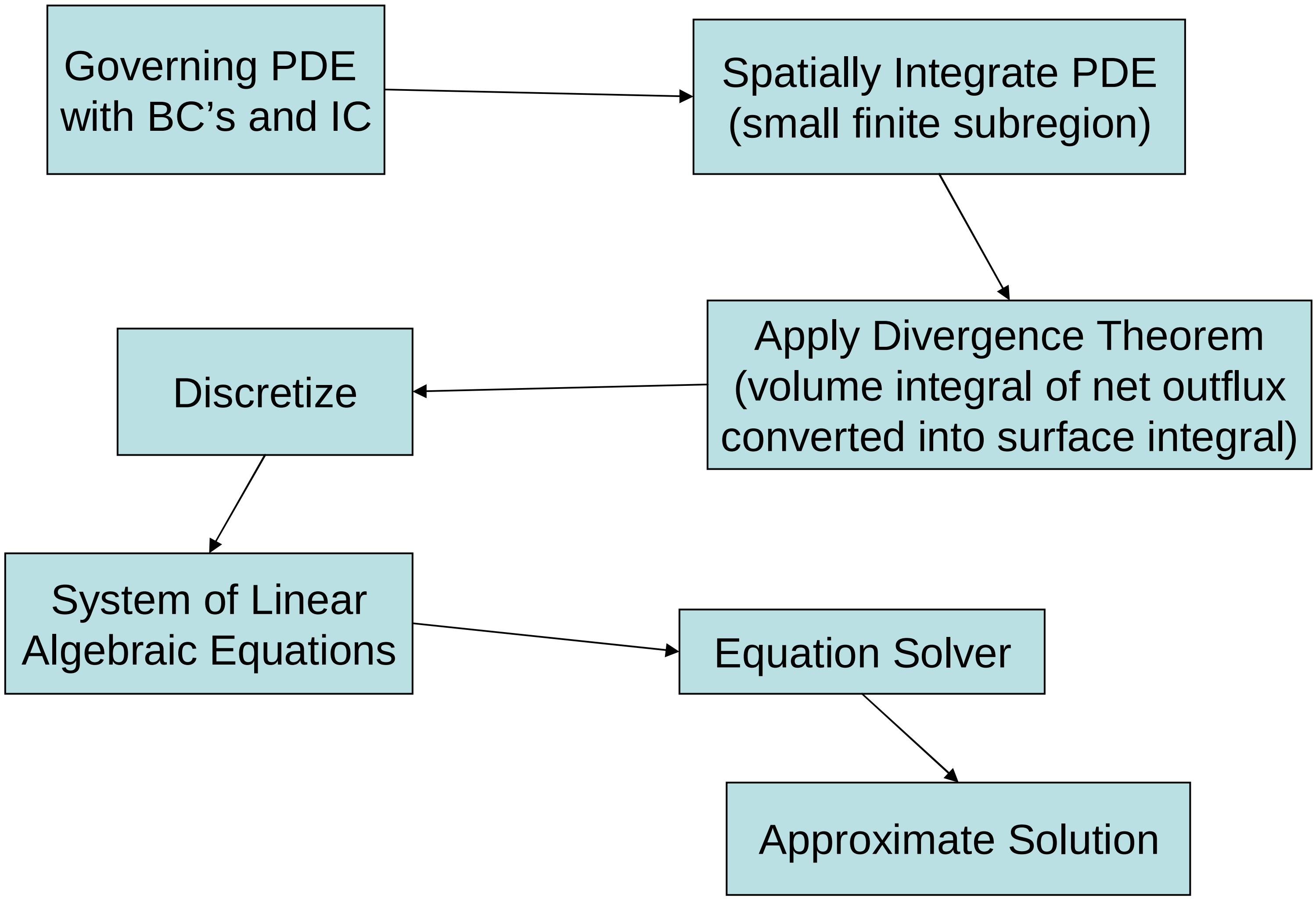
$$K_x \frac{\partial^2 h}{\partial x^2} + K_y \frac{\partial^2 h}{\partial y^2} + K_z \frac{\partial^2 h}{\partial z^2} + q = S_s \frac{dh}{dt} \qquad \left[\frac{1}{T}\right]$$

Discrete Finite-Difference Formulation

$$K_x \frac{h_{i,j-1,k} - 2h_{i,j,k} + h_{i,j+1,k}}{(\Delta x)^2} + K_y \frac{h_{i-1,j,k} - 2h_{i,j,k} + h_{i+1,j,k}}{(\Delta y)^2} +$$

$$K_z \frac{h_{i,j,k-1} - 2h_{i,j,k} + h_{i,j,k+1}}{(\Delta z)^2} + q_{i,j,k} = Ss_{i,j,k} \frac{h^m_{i,j,k} - h^{m-1}_{i,j,k}}{t_m - t_{m-1}} \qquad \left[\frac{1}{T}\right]$$

INTEGRATED FINITE-DIFFERENCE DISCRETIZATION



Governing Flow Equation

$$\frac{\partial}{\partial x}\left(K_x \frac{\partial h}{\partial x}\right)+\frac{\partial}{\partial y}\left(K_y \frac{\partial h}{\partial y}\right)+\frac{\partial}{\partial z}\left(K_z \frac{\partial h}{\partial z}\right)+q=S_s \frac{dh}{dt} \qquad \left[\frac{1}{T}\right]$$

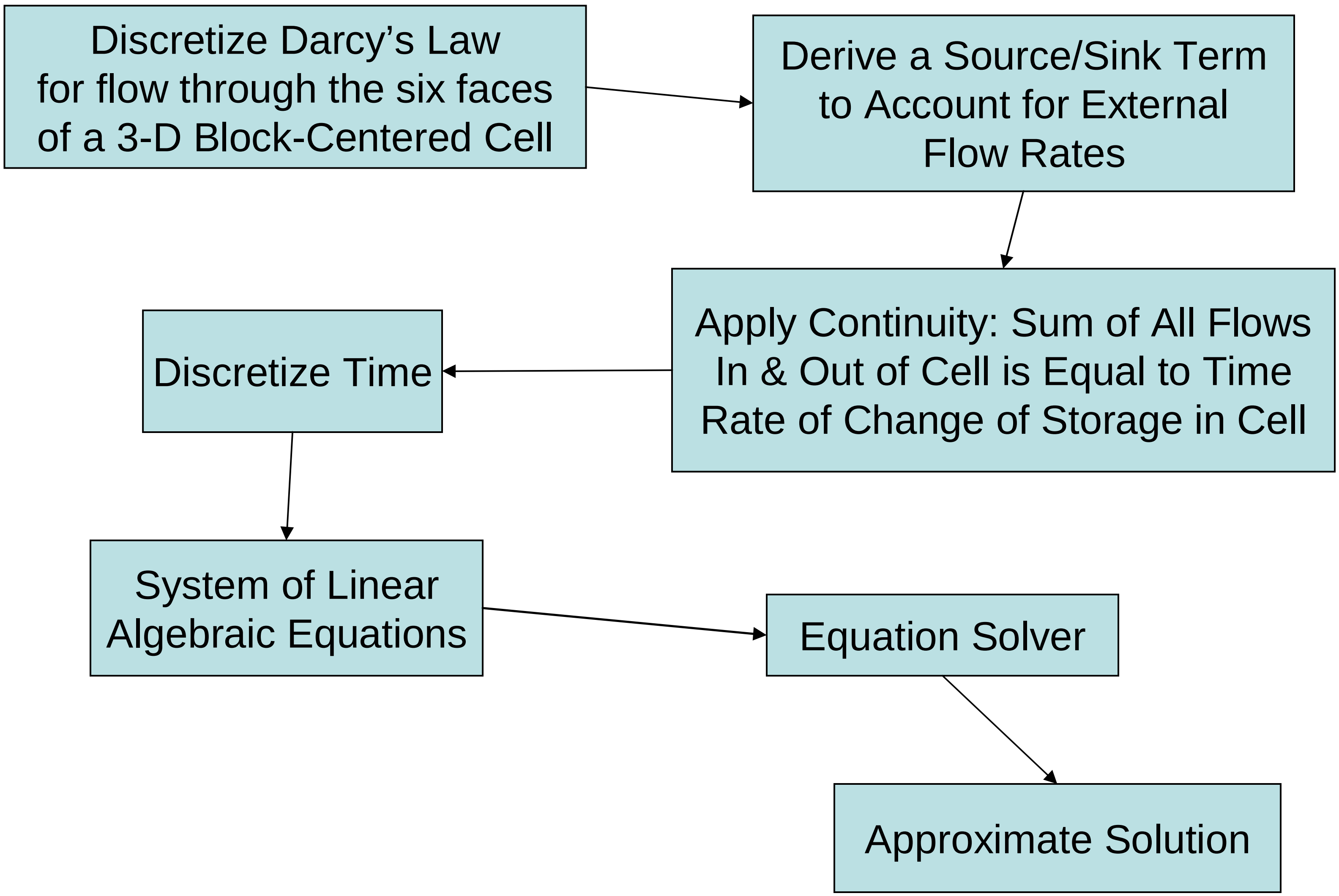
Spatially integrate, apply Divergence Theorem & discretize to get

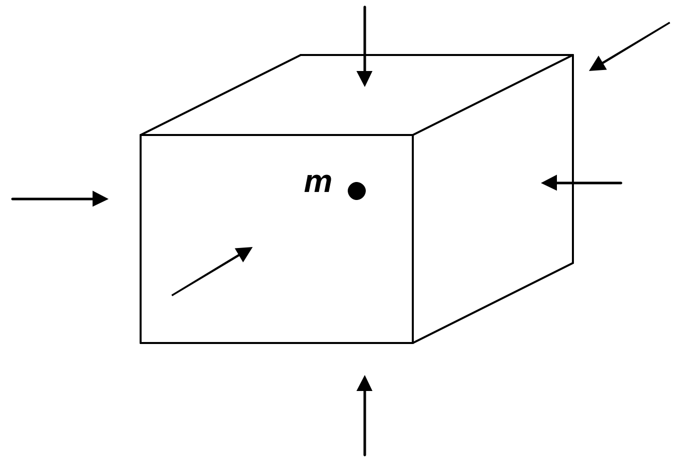
$$\sum_{n=1}^{c} K_{mn} A_{mn} \frac{h_n - h_m}{L_{mn}} + q_m V_m = Ss_m V_m \frac{h_m^k - h_m^{k-1}}{\Delta t}$$

If $C = \frac{KA}{L}$, then

$$\sum_{n=1}^{c} C_{mn}\left(h_n - h_m\right) + q_m V_m = Ss_m V_m \frac{h_m^k - h_m^{k-1}}{\Delta t} \qquad \left[\frac{L^3}{T}\right]$$

MODFLOW'S DISCRETIZATION





Discretize Darcy's Law, derive source/sink term & apply continuity to get

$$\sum_{n=1}^{6} C_{mn}\left(h_n^k - h_m^k\right) + P_m h_m^k + Q_m = Ss_m V_m \frac{h^k - h^{k-1}}{t_k - t_{k-1}} \qquad \left[\frac{L^3}{T}\right]$$

SUMMARY OF DISCRETIZATION SCHEMES

Finite-Difference

$$K_x \frac{h_{i,j-1,k} - 2h_{i,j,k} + h_{i,j+1,k}}{(\Delta x)^2} + K_y \frac{h_{i-1,j,k} - 2h_{i,j,k} + h_{i+1,j,k}}{(\Delta y)^2} +$$

$$K_z \frac{h_{i,j,k-1} - 2h_{i,j,k} + h_{i,j,k+1}}{(\Delta z)^2} + q_{i,j,k} = Ss_{i,j,k} \frac{h_{i,j,k}^{m} - h_{i,j,k}^{m-1}}{t_m - t_{m-1}} \qquad \left[\frac{1}{T}\right]$$

Integrated Finite-Difference

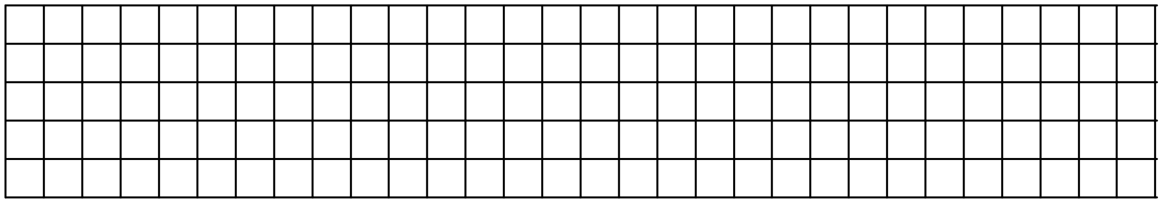
$$\sum_{n=1}^{c} C_{mn}\left(h_n - h_m\right) + q_m V_m = Ss_m V_m \frac{h_m^k - h_m^{k-1}}{\Delta t} \qquad \left[\frac{L^3}{T}\right]$$

MODFLOW

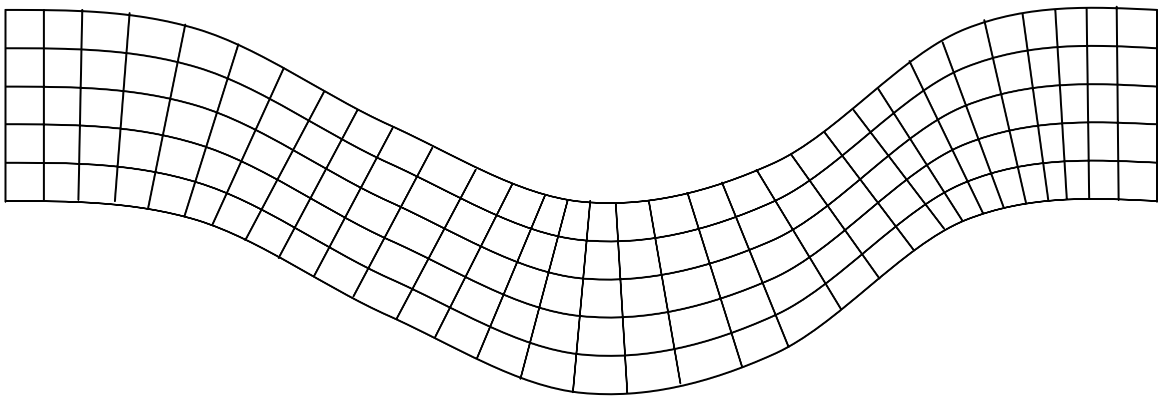
$$\sum_{n=1}^{6} C_{mn}\left(h_n^k - h_m^k\right) + P_m h_m^k + Q_m = Ss_m V_m \frac{h^k - h^{k-1}}{t_k - t_{k-1}} \qquad \left[\frac{L^3}{T}\right]$$

IMPLICATION OF IFD NUMERICAL SCHEME

Finite-Difference Grid



Non-Generalized Integrated Finite-Difference Grid



ENABLING MODFLOW TO UTILIZE ITS IFD METHOD

- Calculate Cell Areas
- Two-Dimensional DELR and DELC arrays
- Adjust Conductance Calculation

ADJUSTMENT TO CONDUCTANCE CALCULATION

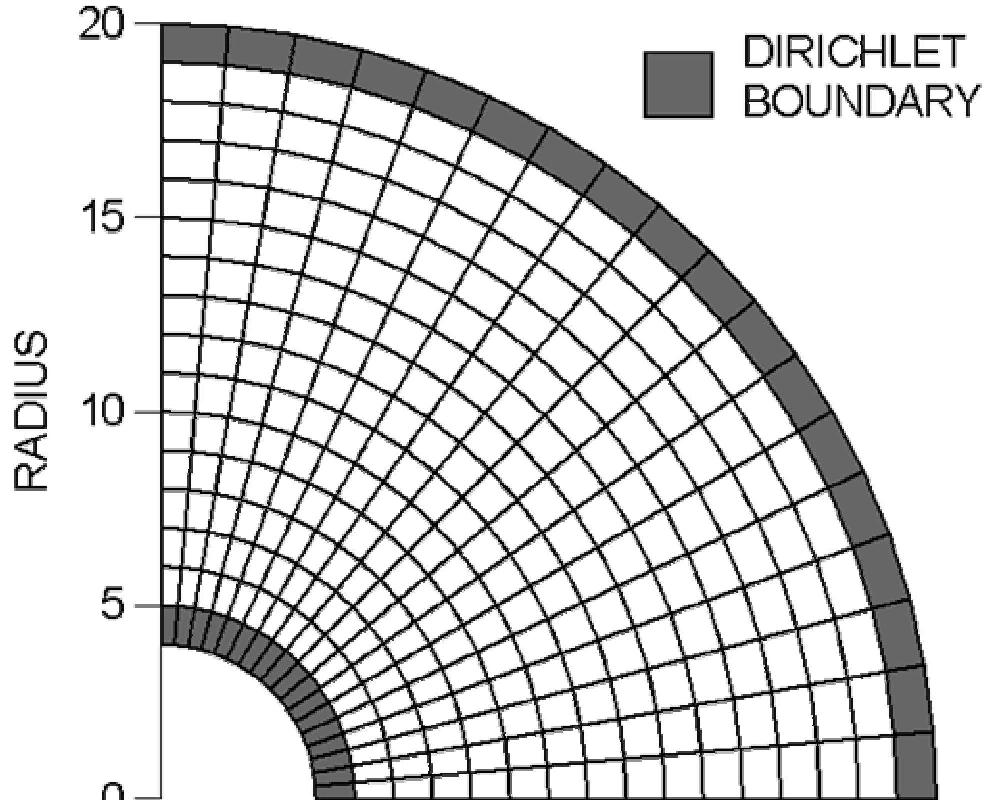
Reduced Form of Conductance Calculation

$$C_{ij} = \frac{2T_i T_j w}{T_i L_j + T_j L_i}$$

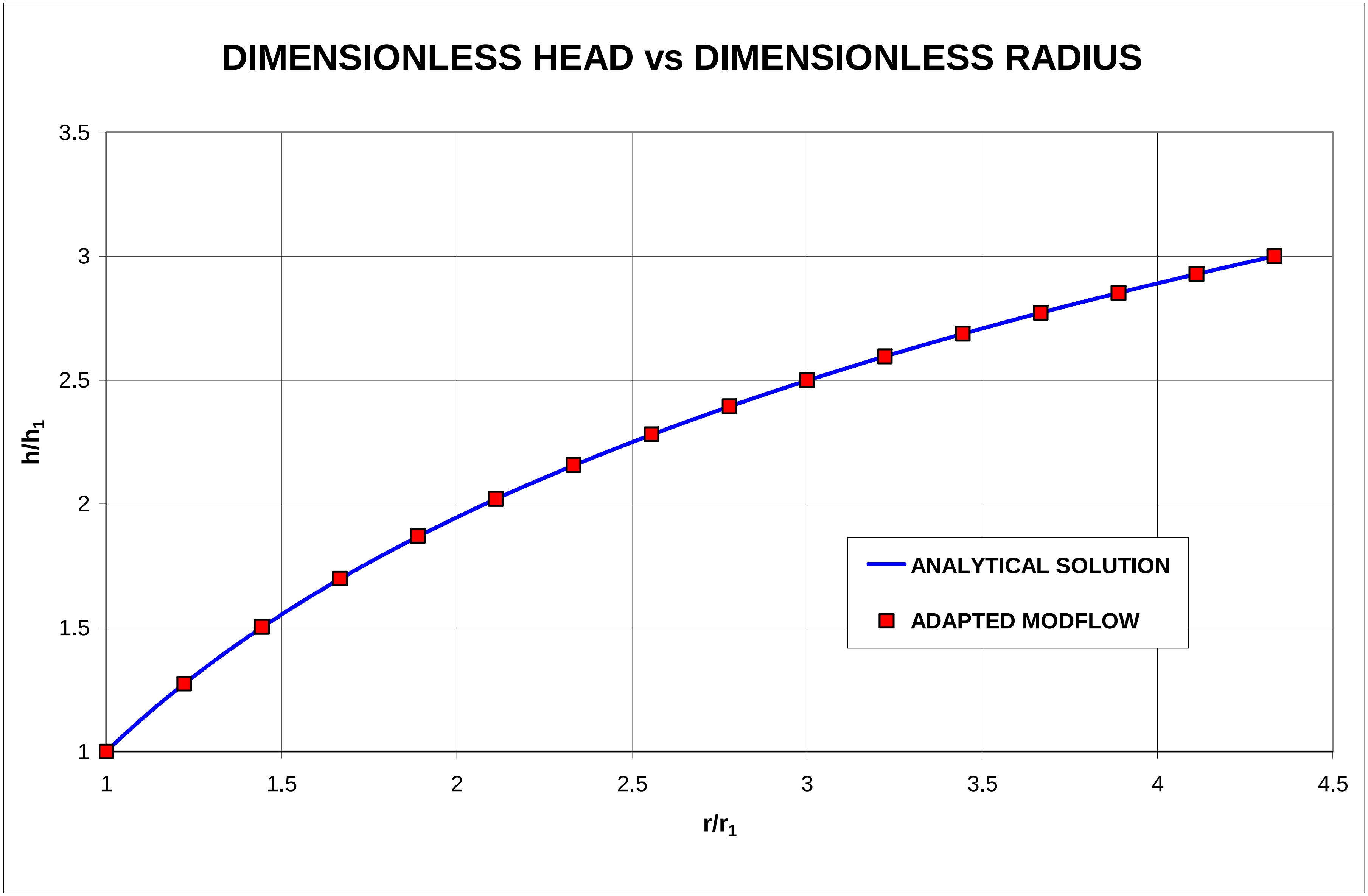
Unreduced Form of Conductance Calculation

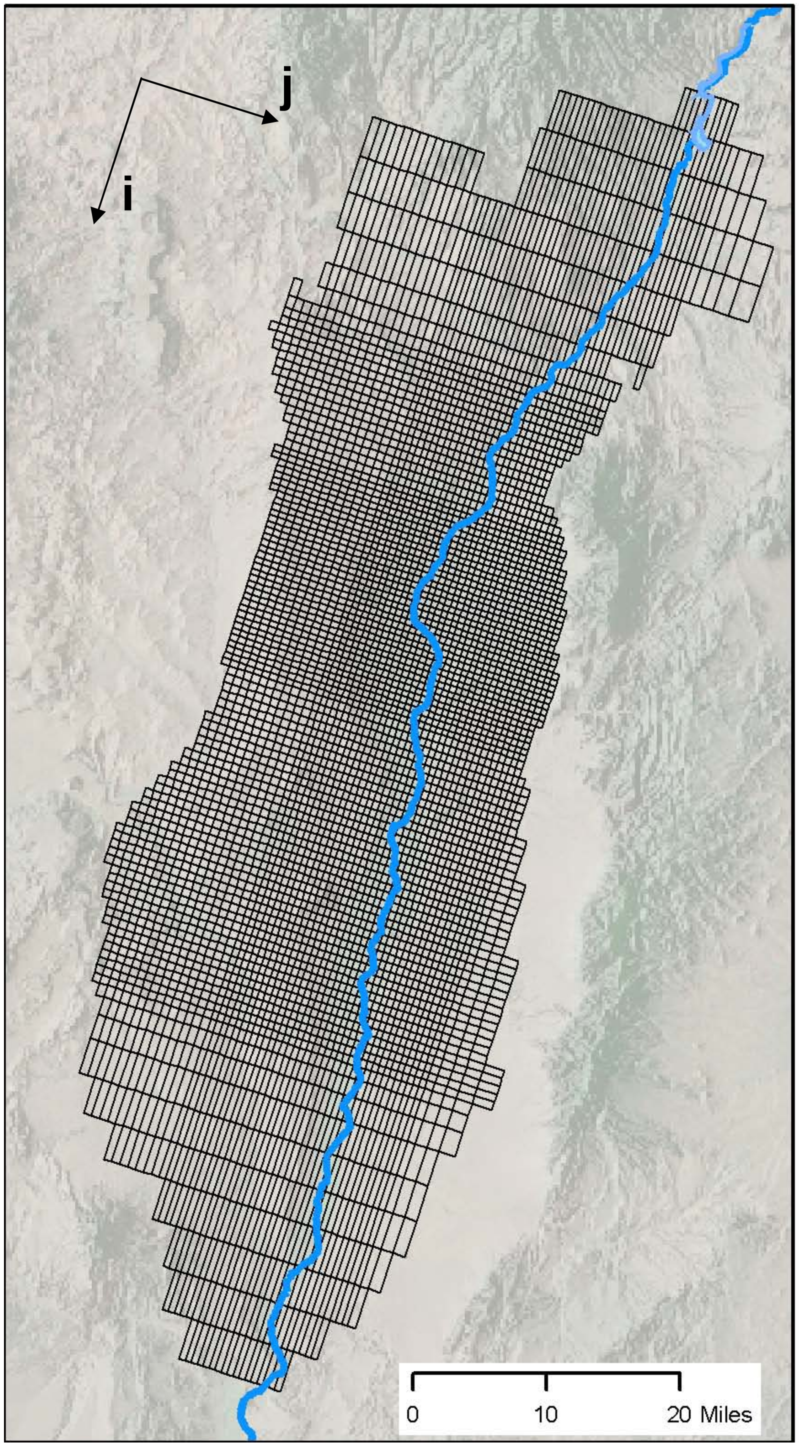
$$C_{ij} = \frac{2T_i w_i T_j w_j}{T_i w_i L_j + T_j w_j L_i}$$

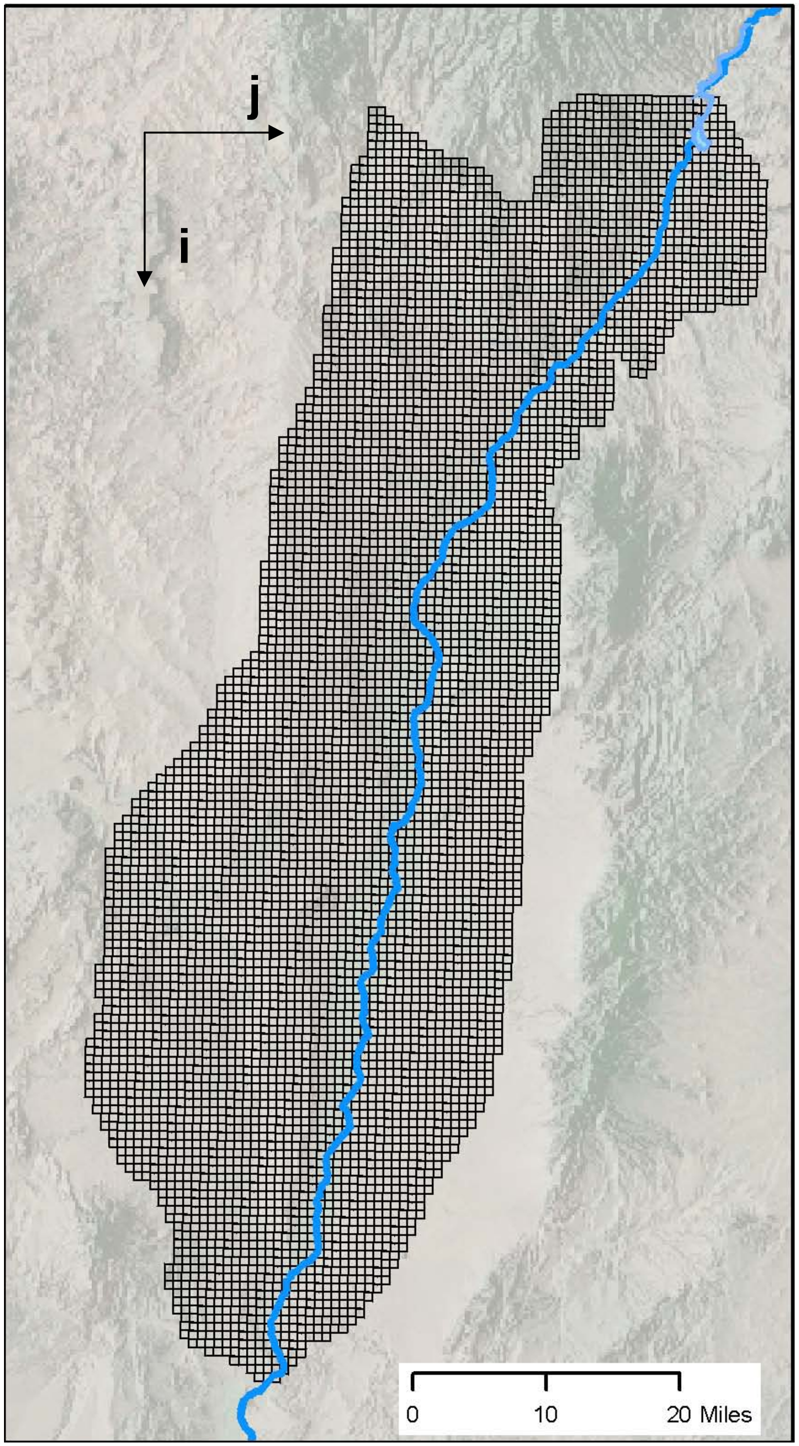
STEADY-STATE RADIAL FLOW PROBLEM



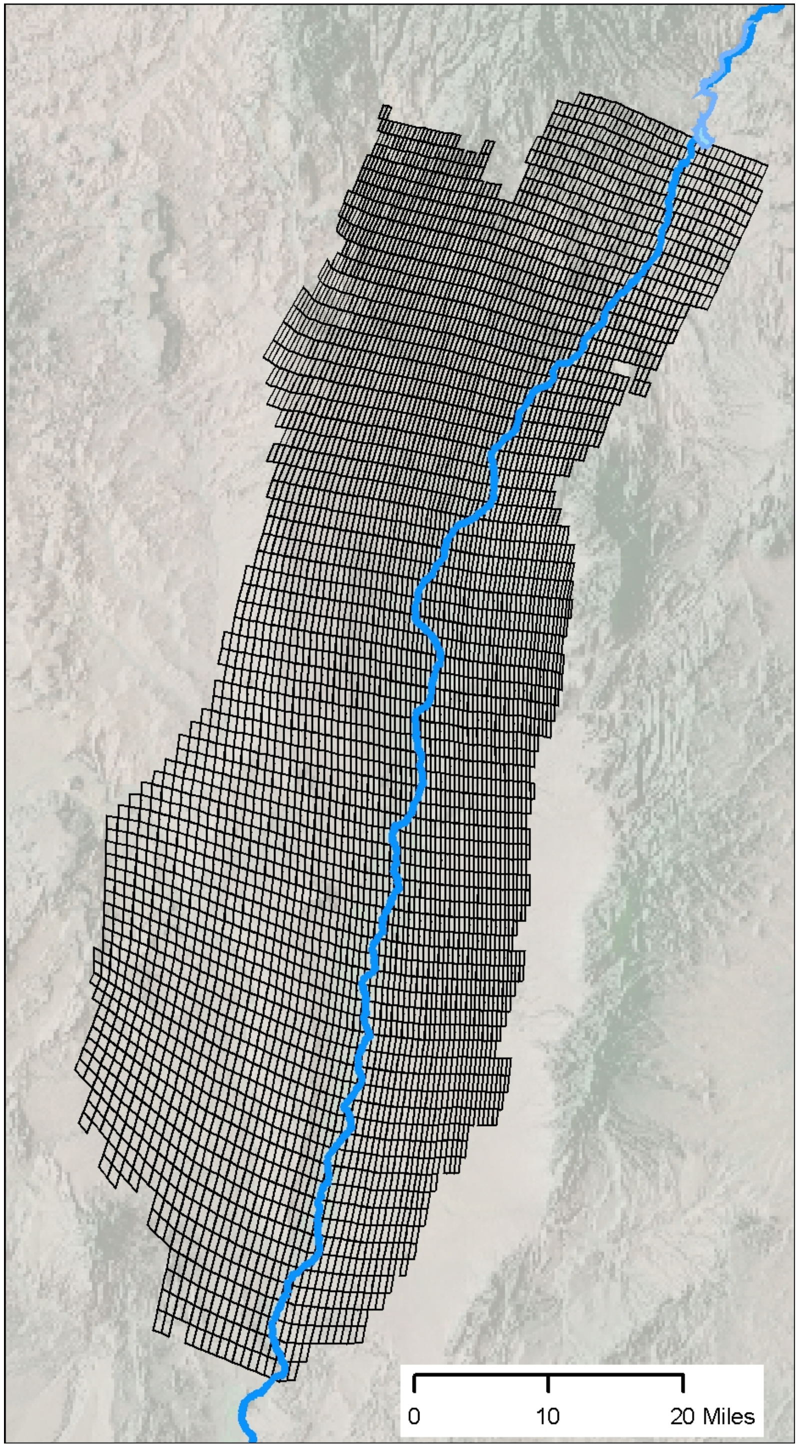
h_1=5 ft h_2=15 ft


DIMENSIONLESS HEAD vs DIMENSIONLESS RADIUS
3.5
3
2.5
2
1.5
1
h/h1
1
1.5
2
2.5
3
3.5
4
4.5
r/r1
ANALYTICAL SOLUTION
ADAPTED MODFLOW


j
i
0
10
20 Miles


j
i
0
10
20 Miles

MODFLOW Grid with IFD Adaptations



CONCLUSIONS

- MODFLOW implements a non-generalized IFD numerical scheme within the confines of a finite difference grid.
- Minor modifications can be made to MODFLOW’s source code to enable flow simulations through a curvilinear grid constructed from trapezoidal cells.
- The modifications maintain compatibility with other MODFLOW packages and increase the versatility of grid construction.